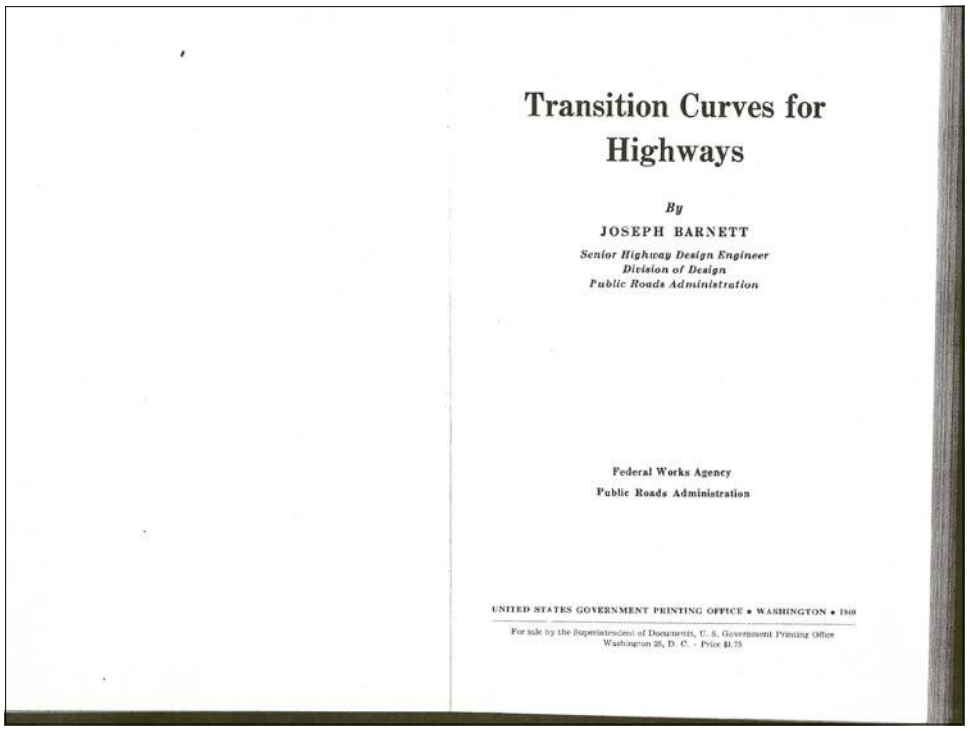
SPIRAL CURVES

A PRACTICAL SOLUTION

Lynn Caswell, L.S. 12/27/08

Transition curves, also known as spiral curves, were used on Interstate Highways, County Roads and Railroads. Most of the public right of ways have been re-platted to eliminate the spirals on right of way but there still are a few to give the Land Surveyor a challenge.

This proposed solution deals with a railroad right of way in Rogers, Minnesota. The example was chosen because our survey involved two different plats and a boundary survey prepared by others. The spiral curve was either ignored or dealt with incorrectly. The tables used for the data are for highway spirals but the application is close enough to give a valid boundary for the survey. If data tables could be found that applied to railroad curves, the calculations and path to the solution would be very similar. The tables, formulas and calculations are from the book "Transition Curves for Highways" by the Federal Works Agency, published in 1940. Copies are available through Amazon.com for a very reasonable price.



Transition Curves for Highways

By

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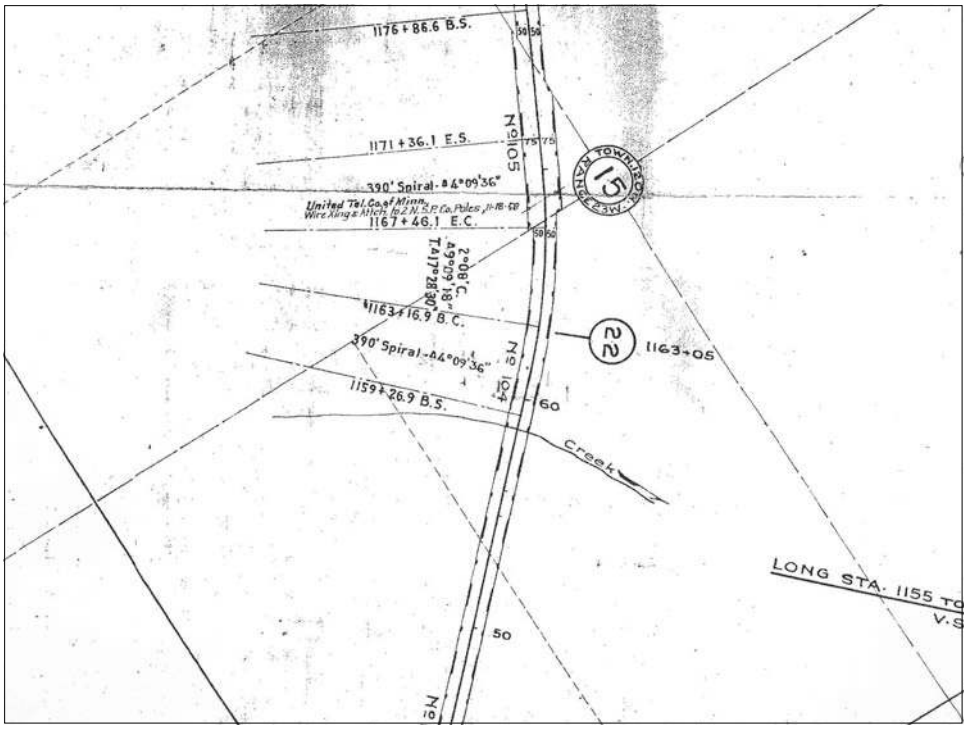
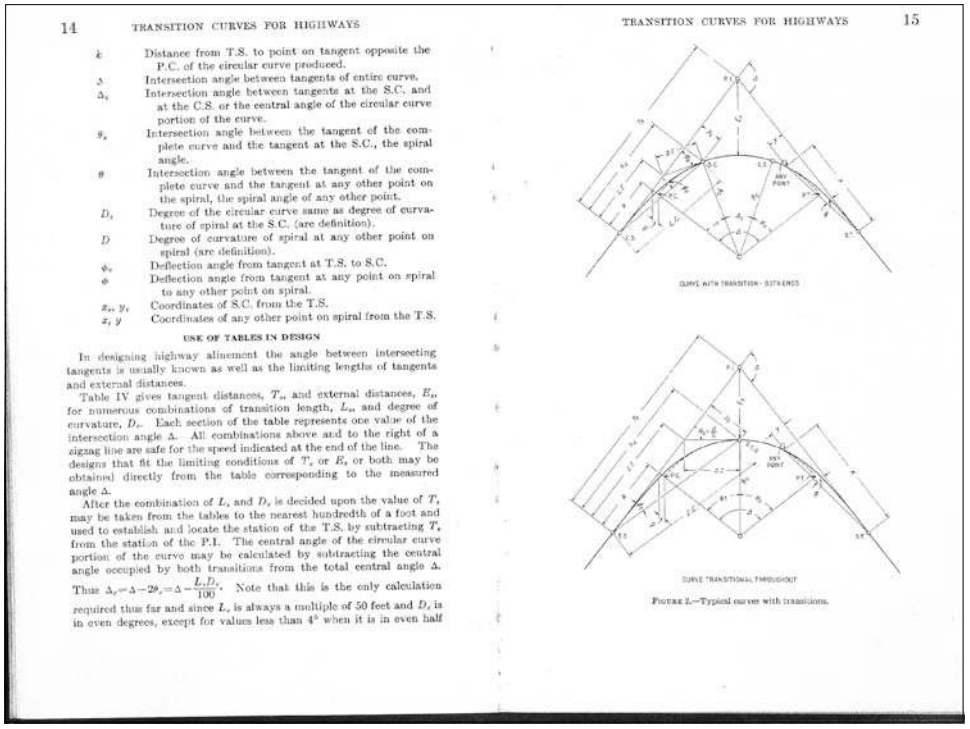
A normal spiral curve consists of three basic parts; the beginning spiral, a central circular curve and ending spiral. Whether working with a highway or a railroad, the right of way map or centerline description is needed. The minimum data necessary is the degree of curve, central angle of the circular curve, length of spiral and spiral central angle. From field measurements and a few calculations the spiral can be positioned.



Locate the center of track from about 500 feet before the curves to 500 feet beyond the curves. A shot every 50 feet through the curves will help to test the theoretical solution to the actual location of the tracks. Location of all found monuments and other physical objects should be included in the survey.

Calculated data needed to set up the curves include the central circular curve radius, circular curve central angle, circular curve tangent, spiral long tangent and spiral short tangents. See the typical curve drawing in the attached data.



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k Distance from T.S. to point on tangent opposite the P.C. of the circular curve produced.
Δ Intersection angle between tangents of entire curve.
Δ_c Intersection angle between tangents at the S.C. and at the C.S. or the central angle of the circular curve portion of the curve.
θ_s Intersection angle between the tangent of the complete curve and the tangent at the S.C., the spiral angle.
θ Intersection angle between the tangent of the complete curve and the tangent at any other point on the spiral, the spiral angle of any other point.
D_c Degree of the circular curve same as degree of curvature of spiral at the S.C. (arc definition).
D Degree of curvature of spiral at any other point on spiral (arc definition).
ϕ_s Deflection angle from tangent at T.S. to S.C.
ϕ Deflection angle from tangent at any point on spiral to any other point on spiral.
x_s, y_s Coordinates of S.C. from the T.S.
x, y Coordinates of any other point on spiral from the T.S.

USE OF TABLES IN DESIGN

In designing highway alinement the angle between intersecting tangents is usually known as well as the limiting lengths of tangents and external distances.

Table IV gives tangent distances, T_s, and external distances, E_s, for numerous combinations of transition length, L_s, and degree of curvature, D_c. Each section of the table represents one value of the intersection angle Δ. All combinations above and to the right of a zigzag line are safe for the speed indicated at the end of the line. The designs that fit the limiting conditions of T_s or E_s, or both may be obtained directly from the table corresponding to the measured angle Δ.

After the combination of L_s and D_c is decided upon the value of T_s may be taken from the tables to the nearest hundredth of a foot and used to establish and locate the station of the T.S. by subtracting T_s from the station of the P.I. The central angle of the circular curve portion of the curve may be calculated by subtracting the central angle occupied by both transitions from the total central angle Δ. Thus $\Delta_c = \Delta - 2\theta_s = \Delta - \frac{L_s D_c}{100}$. Note that this is the only calculation required thus far and since L_s is always a multiple of 50 feet and D_c is in even degrees, except for values less than 4° when it is in even half

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FIGURE 2.—Typical curves with transitions.

My assumption is that most surveyors will use cad software for the calculations. The solution can be tested manually. The method is in the above-mentioned book.

Use the field data to establish a best fit of the tangents running into and out of the curves. Intersect the tangents to determine P.I. of the curve set.

Determine the overall central angle for the curve set. The overall central angle will rarely match the plan angle. Adjust the difference in the circular curve and hold the spiral curve angles to the map or description angles.

Overall Central Angle	17 d 26’ 06”
- Spiral Angle	-04 d 09’ 36”
“	-04 d 09’ 36”
Circular Central Angle	09 d 06’ 54”

This application is for a rail line so the chord definition is used to calculate the radius of the circular curve.

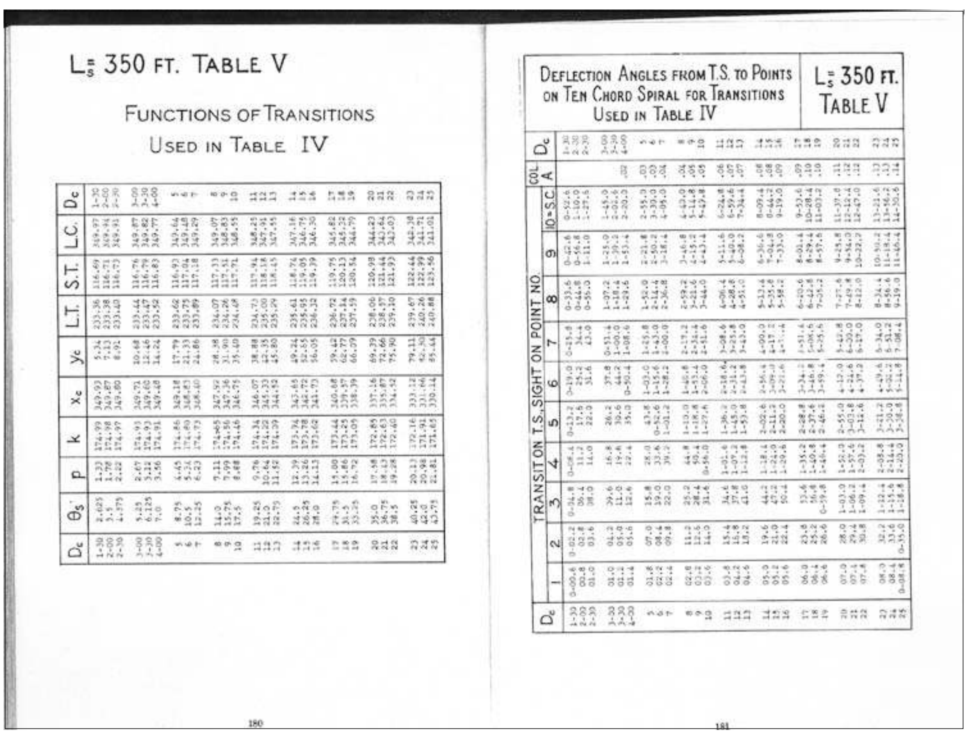
Radius = 50/Sin(2 d 08’/2) = 2685.89

Calculate the Tangent for the circular curve.

Tangent = 2685.89 * Tan (09 d 06’ 54”/2) = 214.11

All highway spirals are referred to as 10 chord spirals. The length is always a multiple of 50 feet. Railroad spirals do not hold to the length rule so their needs to be some interpolation of the tables to achieve a reasonable solution.

Use the tables to determine the Long Tangent and Short Tangent for a 390 foot spiral. Use the data for a 2d 00’ curve or interpolate if the variation is large enough to make a difference.



L_s = 350 FT. TABLE V

FUNCTIONS OF TRANSITIONS USED IN TABLE IV

D_c	θ_s	p	k	x_c	y_c	L.T.	S.T.	L.C.	D_c
1-30	2.625	1.33	174.99	349.93	5.34	233.36	116.69	349.97	1-30
2-00	3.5	1.78	174.98	349.87	7.13	233.38	116.71	349.94	2-00
2-30	4.375	2.22	174.97	349.80	8.91	233.40	116.73	349.91	2-30
3-00	5.25	2.67	174.93	349.71	10.68	233.44	116.76	349.87	3-00
3-30	6.125	3.11	174.93	349.60	12.46	233.47	116.79	349.82	3-30
4-00	7.0	3.56	174.91	349.48	14.24	233.52	116.83	349.77	4-00
5	8.75	4.45	174.86	349.18	17.79	233.62	116.93	349.64	5
6	10.5	5.34	174.80	348.83	21.33	233.75	117.04	349.48	6
7	12.25	6.23	174.73	348.40	24.86	233.89	117.18	349.29	7
8	14.0	7.11	174.65	347.92	28.38	234.07	117.33	349.07	8
9	15.75	7.99	174.56	347.36	31.90	234.26	117.51	348.83	9
10	17.5	8.88	174.46	346.75	35.40	234.48	117.71	348.55	10
11	19.25	9.76	174.34	346.07	38.88	234.73	117.94	348.25	11
12	21.0	10.64	174.22	345.33	42.35	235.00	118.18	347.91	12
13	22.75	11.52	174.09	344.52	45.80	235.29	118.45	347.55	13
14	24.5	12.39	173.94	343.65	49.24	235.61	118.74	347.16	14
15	26.25	13.26	173.78	342.72	52.65	235.95	119.05	346.75	15
16	28.0	14.13	173.62	341.73	56.05	236.32	119.39	346.30	16
17	29.75	15.00	173.44	340.68	59.42	236.72	119.75	345.82	17
18	31.5	15.86	173.25	339.57	62.77	237.14	120.13	345.32	18
19	33.25	16.72	173.05	338.39	66.09	237.59	120.54	344.79	19
20	35.0	17.58	172.85	337.16	69.39	238.06	120.98	344.23	20
21	36.75	18.43	172.63	335.87	72.66	238.57	121.44	343.64	21
22	38.5	19.28	172.40	334.52	75.90	239.10	121.93	343.03	22
23	40.25	20.13	172.16	333.12	79.11	239.67	122.44	342.38	23
24	42.0	20.98	171.91	331.66	82.30	240.26	122.99	341.71	24
25	43.75	21.81	171.65	330.14	85.44	240.88	123.56	341.01	25

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DEFLECTION ANGLES FROM T.S. TO POINTS ON TEN CHORD SPIRAL FOR TRANSITIONS USED IN TABLE IV

L_s = 350 FT. TABLE V

D_c	1	2	3	4	5	6	7	8	9	10=S.C.	COL. A	D_c
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TRANSIT ON T.S., SIGHT ON POINT NO.

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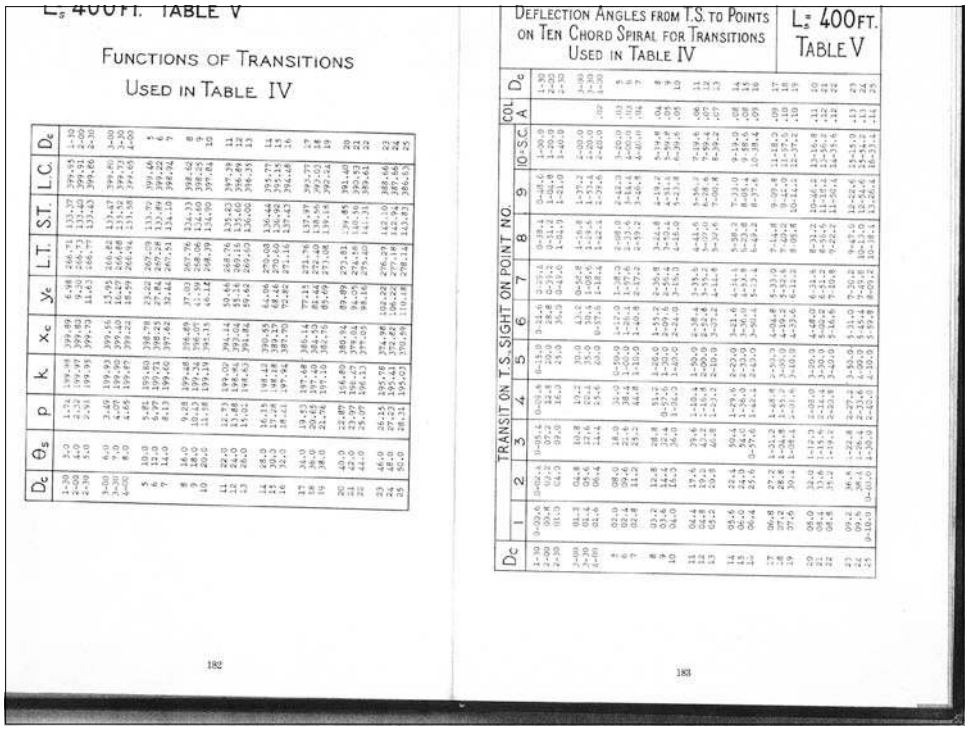


Short Tangent (ST) 350' Spiral 116.71
Short Tangent (ST) 400' Spiral 133.40

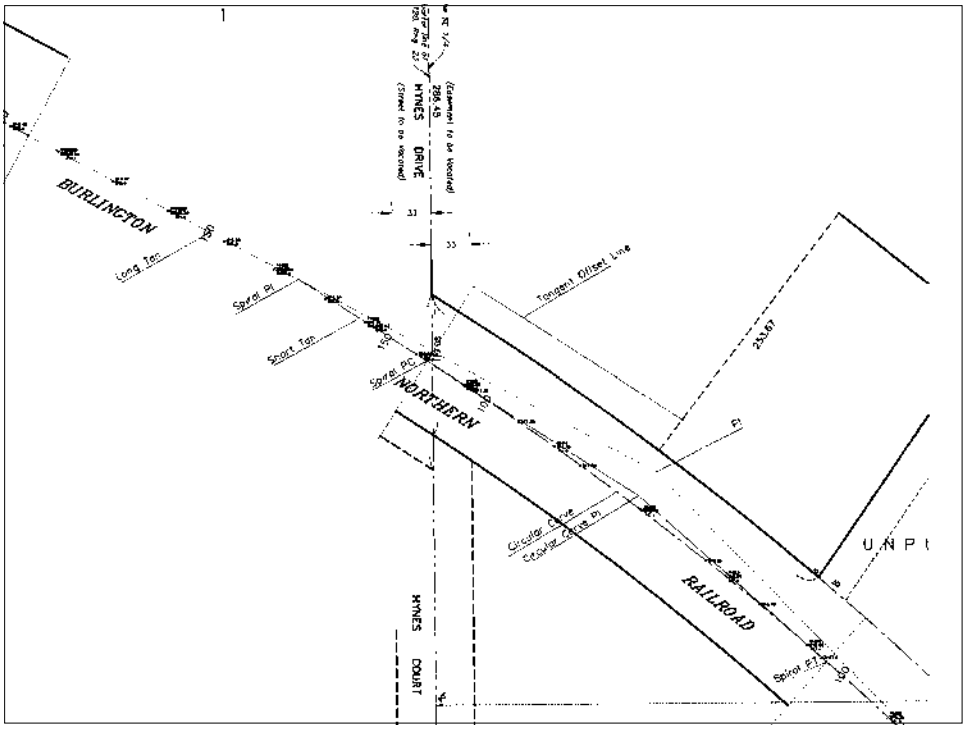
40/50 * (133.40-116.71) + 116.71 = 130.06

Long Tangent (LT) 350' Spiral 233.38
Long Tangent (LT) 400' Spiral 266.73

40/50 * (266.73-233.38) + 233.38 = 260.06

Off to the side in the cad software, use the tangent data and central angles to set up the curve tangents. Move the tangents onto the drawing, matching lines of the track centerline.

Extend radial lines to the right of way for the Begin Spiral, Spiral Circular point (SC), Circular Spiral point (CS) and Spiral Tangent (ST).



Construct the spiral on centerline using cad.

Curves and Lines
Create Spirals
Fit Tangent to Curve
Select Tangent
Select Curve

Zoom into the centerline to test the physical location of the tracks relative to the calculated curves. The expectation is that the mathematical solution should be within a few tenths of a foot of the actual location of the tracts. If there is a greater divergence, then the curves should be adjusted for a better fit.

Offset the Circular Curve to set the right of way.

There are two ways to establish the spiral curve on the right of way. The first, is to simply offset it the same as the circular curve. It's a reasonable solution and width checks through the curve indicate that is a parallel spiral. If the curve is measured using cad inquiry, the data returned is the same as centerline.

The second solution, is to use the method outlined above for the spiral on a centerline. The solution will give a different curve than the offset method and the width will also check properly. There will be some variation in the locations of the TS, SC, CS and ST. These positions can be adjusted by trimming and extending the curves to the original positions established by extending the radial lines. It is a good idea to calculate areas before trimming and adjusting the locations of the TS, SC, CS and ST.

The spiral curve data to be shown for the survey or plat should be the chord and chord bearing. The chord bearing and chord length can be measured directly from the location of the TS and SC. The Spiral Length is determined from the Table II - Functions of Transitions.

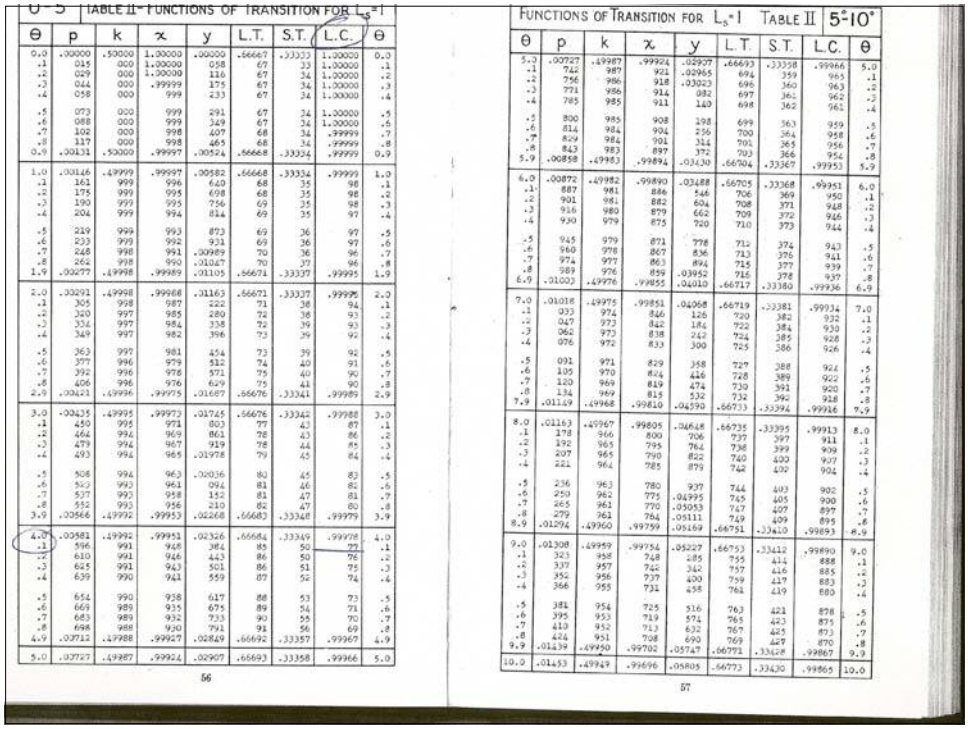


TABLE II- FUNCTIONS OF TRANSITION FOR $L_s=1$

θ	p	k	x	y	L.T.	S.T.	L.C.	θ
0.0	.00000	.50000	1.00000	.00000	.66667	.33333	1.00000	0.0
.1	015	000	1.00000	058	67	33	1.00000	.1
.2	029	000	1.00000	116	67	34	1.00000	.2
.3	044	000	.99999	175	67	34	1.00000	.3
.4	058	000	999	233	67	34	1.00000	.4
.5	073	000	999	291	67	34	1.00000	.5
.6	088	000	999	349	67	34	1.00000	.6
.7	102	000	998	407	68	34	.99999	.7
.8	117	000	998	465	68	34	.99999	.8
0.9	.00131	.50000	.99997	.00524	.66668	.33334	.99999	0.9
1.0	.00146	.49999	.99997	.00582	.66668	.33334	.99999	1.0
.1	161	999	996	640	68	35	98	.1
.2	175	999	995	698	68	35	98	.2
.3	190	999	995	756	69	35	98	.3
.4	204	999	994	814	69	35	97	.4
.5	219	999	993	873	69	36	97	.5
.6	233	999	992	931	69	36	97	.6
.7	248	998	991	.00989	70	36	96	.7
.8	262	998	990	.01047	70	37	96	.8
1.9	.00277	.49998	.99989	.01105	.66671	.33337	.99995	1.9
2.0	.00291	.49998	.99988	.01163	.66671	.33337	.99995	2.0
.1	305	998	987	222	71	38	94	.1
.2	320	997	985	280	72	38	93	.2
.3	334	997	984	338	72	39	93	.3
.4	349	997	982	396	73	39	92	.4
.5	363	997	981	454	73	39	92	.5
.6	377	996	979	512	74	40	91	.6
.7	392	996	978	571	75	40	90	.7
.8	406	996	976	629	75	41	90	.8
2.9	.00421	.49996	.99975	.01687	.66676	.33341	.99989	2.9
3.0	.00435	.49995	.99972	.01745	.66676	.33342	.99988	3.0
.1	450	995	971	803	77	43	87	.1
.2	464	994	969	861	78	43	86	.2
.3	479	994	967	919	78	44	85	.3
.4	493	994	965	.01978	79	45	84	.4
.5	508	994	963	.02036	80	45	83	.5
.6	522	993	961	094	81	46	82	.6
.7	537	993	958	152	81	47	81	.7
.8	552	993	956	210	82	47	80	.8
3.9	.00566	.49992	.99953	.02268	.66683	.33348	.99979	3.9
4.0	.00581	.49992	.99951	.02326	.66684	.33349	.99978	4.0
.1	596	991	948	384	85	50	77	.1
.2	610	991	946	443	86	50	76	.2
.3	625	991	943	501	86	51	75	.3
.4	639	990	941	559	87	52	74	.4
.5	654	990	938	617	88	53	73	.5
.6	669	989	935	675	89	54	71	.6
.7	683	989	932	733	90	55	70	.7
.8	698	988	930	791	91	56	69	.8
4.9	.00712	.49988	.99927	.02849	.66692	.33357	.99967	4.9
5.0	.00727	.49987	.99924	.02907	.66693	.33358	.99966	5.0

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FUNCTIONS OF TRANSITION FOR $L_s=1$ TABLE II 5°-10°

θ	p	k	x	y	L.T.	S.T.	L.C.	θ
5.0	.00727	.49987	.99924	.02907	.66693	.33358	.99966	5.0
.1	742	987	921	.02965	694	359	965	.1
.2	756	986	918	.03023	696	360	963	.2
.3	771	986	914	082	697	361	962	.3
.4	785	985	911	140	698	362	961	.4
.5	800	985	908	198	699	363	959	.5
.6	814	984	904	256	700	364	958	.6
.7	829	984	901	314	701	365	956	.7
.8	843	983	897	372	703	366	954	.8
5.9	.00858	.49983	.99894	.03430	.66704	.33367	.99953	5.9
6.0	.00872	.49982	.99890	.03488	.66705	.33368	.99951	6.0
.1	887	981	886	546	706	369	950	.1
.2	901	981	882	604	708	371	948	.2
.3	916	980	879	662	709	372	946	.3
.4	930	979	875	720	710	373	944	.4
.5	945	979	871	778	712	374	943	.5
.6	960	978	867	836	713	375	941	.6
.7	974	977	863	894	715	377	939	.7
.8	989	976	859	.03952	716	378	937	.8
6.9	.01003	.49976	.99855	.04010	.66717	.33380	.99936	6.9
7.0	.01018	.49975	.99851	.04068	.66719	.33381	.99934	7.0
.1	033	974	846	126	720	382	932	.1
.2	047	973	842	184	722	384	930	.2
.3	062	973	838	242	724	385	928	.3
.4	076	972	833	300	725	386	926	.4
.5	091	971	829	358	727	388	924	.5
.6	105	970	824	416	728	389	922	.6
.7	120	969	819	474	730	391	920	.7
.8	134	969	815	532	732	392	918	.8
7.9	.01149	.49968	.99810	.04590	.66733	.33394	.99916	7.9
8.0	.01163	.49967	.99805	.04648	.66735	.33395	.99913	8.0
.1	178	966	800	706	737	397	911	.1
.2	192	965	795	764	738	399	909	.2
.3	207	965	790	822	740	400	907	.3
.4	221	964	785	879	742	402	904	.4
.5	236	963	780	937	744	403	902	.5
.6	250	962	775	.04995	745	405	900	.6
.7	265	961	770	.05053	747	407	897	.7
.8	279	961	764	.05111	749	409	895	.8
8.9	.01294	.49960	.99759	.05169	.66751	.33410	.99893	8.9
9.0	.01308	.49959	.99754	.05227	.66753	.33412	.99890	9.0
.1	323	958	748	285	755	414	888	.1
.2	337	957	742	342	757	416	885	.2
.3	352	956	737	400	759	417	883	.3
.4	366	955	731	458	761	419	880	.4
.5	381	954	725	516	763	421	878	.5
.6	395	953	719	574	765	423	875	.6
.7	410	952	713	632	767	425	873	.7
.8	424	951	708	690	769	427	870	.8
9.9	.01439	.49950	.99702	.05747	.66771	.33428	.99867	9.9
10.0	.01453	.49949	.99696	.05805	.66773	.33430	.99865	10.0

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Spiral Central 4.1 d = LC 0.99977

Spiral Length = Chord / LC = 389.91/0.99977 = 390.00

Overall Spiral curve lengths on the right of way can be determined using the same equation. Arc lengths of sub-portions of spirals can't be determined using this method. For sub-portions of the spiral, show the Chord and Chord Bearing.

Calculating an area can be a problem with cad once the curves have been adjusted. There may be enough of a gap at the curve ends to cause the software to fail. A solution is to draw and use the chords to measure the area. There may need to be an adjustment for the area between the arc and chord.