FITTING TANGENT CIRCULAR CURVES
THROUGH AS-BUILT SPIRAL RIGHTS-OF-WAY

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BIOGRAPHICAL SKETCH

Jack A. Savlan is a cadastral surveyor with the Bureau of
Land Management. Having received his B.S. degree from
Oregon Institute of Technology in 1982, he has since been
working in Arizona and New Mexico. Some of his experi-
ences include original cadastral surveys upon the Navajo
lands, and resurveys of townships. Highway geometry
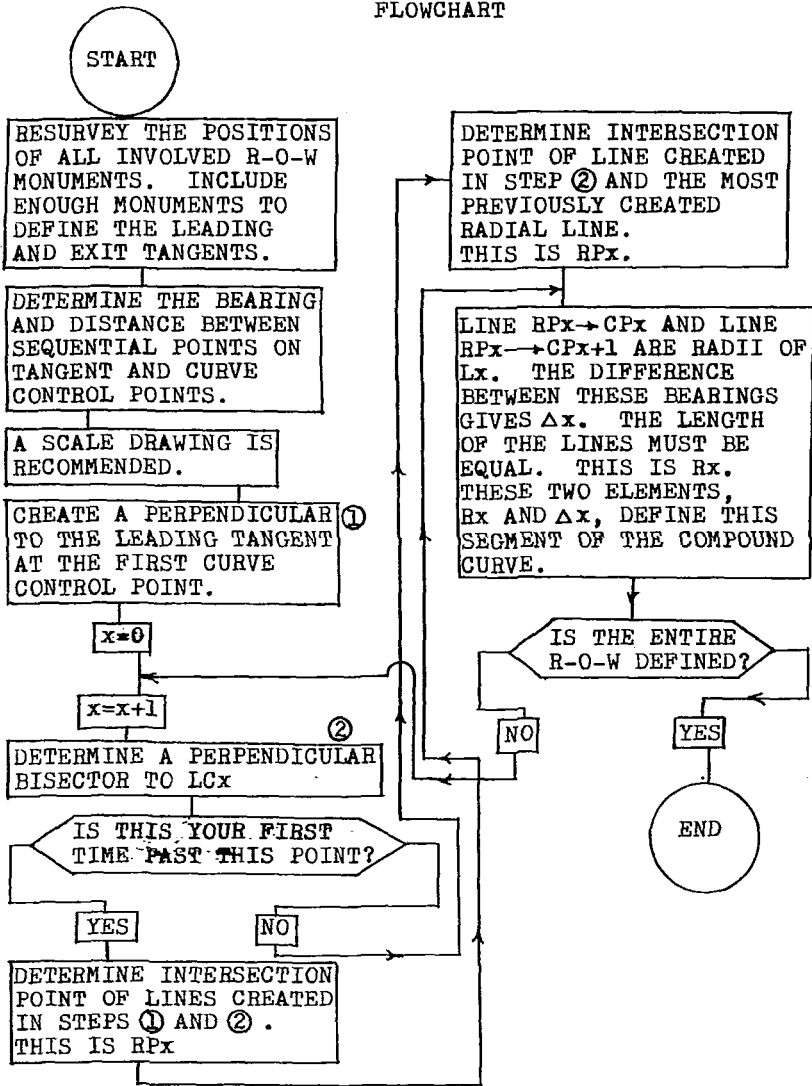remains an area of active interest.

ABSTRACT

Several mathematicians have written high powered algor-
ithums for determining area adjacent to spiral curves.
Indeed these methods are effective when highway data and
ground conditions closely match. Too frequently though,
positions of spiral right-of-way monuments do not remotely
resemble written record. Defining necessary curve data
by creating a new centerline spiral from scattered right-
of-way points is rigorous and sometimes impossible. How
then can a boundary surveyor competently determine the
area of an adjacent parcel? The following solution is
an instructional guide for fitting a series of tangent
circular curves through a series of scattered right-of-
way monuments.

INTRODUCTION

For monuments to control, they must be called on the
appropriate plat, field notes, or deed. Called monuments
are controlling over record distance, bearing, and area.
Uncalled monuments only control when harmonious with other
elements. If highway monuments are not called, the asphalt
may be considered intent, and centerline controls. Only
where no on ground improvement exists may mathematical
dimensions control. Monuments are subordinate to senior
rights, clearly stated contrary intentions, and original
rectangular surveyed lines.

How should a resurveyor handle conflict between highway
centerline, right-of-way monuments, and plat? Every case
is a new ball game. How was the land acquired? Was the
plat created for acquistion, or as-built? Were right-
of-way monuments set before acquisition, during construc-
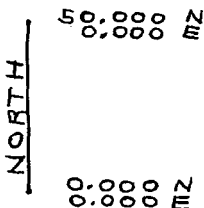tion, or as an afterthought?

START

RESURVEY THE POSITIONS
OF ALL INVOLVED R-O-W
MONUMENTS. INCLUDE
ENOUGH MONUMENTS TO
DEFINE THE LEADING
AND EXIT TANGENTS.

DETERMINE THE BEARING
AND DISTANCE BETWEEN
SEQUENTIAL POINTS ON
TANGENT AND CURVE
CONTROL POINTS.

A SCALE DRAWING IS
RECOMMENDED.

CREATE A PERPENDICULAR ①
TO THE LEADING TANGENT
AT THE FIRST CURVE
CONTROL POINT.

x=0

x=x+1

②
DETERMINE A PERPENDICULAR
BISECTOR TO LCx

IS THIS YOUR FIRST
TIME PAST THIS POINT?

YES   NO

DETERMINE INTERSECTION
POINT OF LINES CREATED
IN STEPS ① AND ② .
THIS IS RPx

DETERMINE INTERSECTION
POINT OF LINE CREATED
IN STEP ② AND THE MOST
PREVIOUSLY CREATED
RADIAL LINE.
THIS IS RPx.

LINE RPx→CPx AND LINE
RPx→CPx+1 ARE RADII OF
Lx. THE DIFFERENCE
BETWEEN THESE BEARINGS
GIVES Δx. THE LENGTH
OF THE LINES MUST BE
EQUAL. THIS IS Rx.
THESE TWO ELEMENTS,
Rx AND Δx, DEFINE THIS
SEGMENT OF THE COMPOUND
CURVE.

IS THE ENTIRE
R-O-W DEFINED?

NO   YES

END

RP=Radius Point
CP=Curve Control Point
L=Curve
R=Radius
LC=Long Chord
Δ=Central Angle

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Resurveying a highway right-of-way, boundary monuments are found at the following relative positions. POT1=0.000N, 0.000E; CP1=50.000N, 0.000E; CP2=347.311N, 29.816E; CP3=676.866N, 359.371E; CP4=706.682N, 656.682E, POT2= 706.682N, 800.000E. Coordinates are in units of feet.
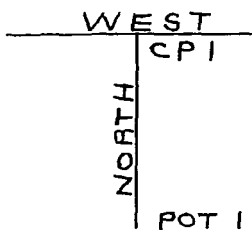
Step 1
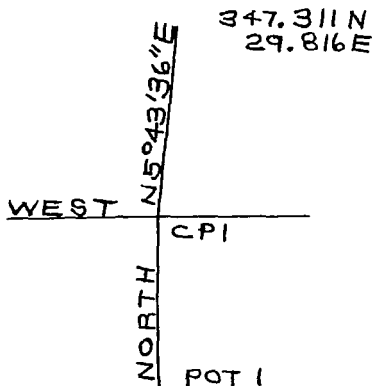Sketch POT1, CP1, and inverse for bearing

```
                    50.000 N
                     0.000 E
         I
         T
         R
         O
         N
                     0.000 N
                     0.000 E
```

Step 2
Strike a perpendicular at CP1 and determine bearing. This is the first radial line.

```
       WEST
              CP 1
          I
          T
          R
          O
          N
              POT 1
```
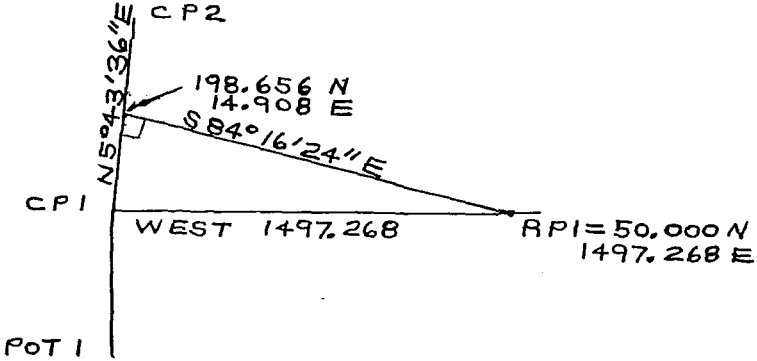
Step 3
Plot CP2. Inverse to CP1. This is the long chord of the first curve.

```
                    347.311 N
                     29.816 E
         N 5°43'36" E
       WEST
              CP1
          I
          T
          R
          O
          N
              POT 1
```

Step 4

From the long chord created in step 3, strike a perpendic-
ular bisector to intersection with the line created in
step 2. Determine the coordinates of intersection. This
is the first radius point. Inverse between CP1 and RP1
for radial length of first curve.

CP2

198.656 N
14.908 E

S 84° 16' 24" E

N 5° 43' 36" E

CP1

WEST   1497.268

RP1 = 50.000 N
1497.268 E

PoT 1

Step 5

Inverse between CP2 and RP1. This determines the second
radial line. Compute the angle between the first and
second radial lines. This is the first central angle.
The first curve is now defined. R=1497.268 ft. Central
Angle=11°27'12"

CP2

N 78° 32' 48" W

CENTRAL ANGLE
= 11° 27' 12"

CP1

R = 1497.268

RP1
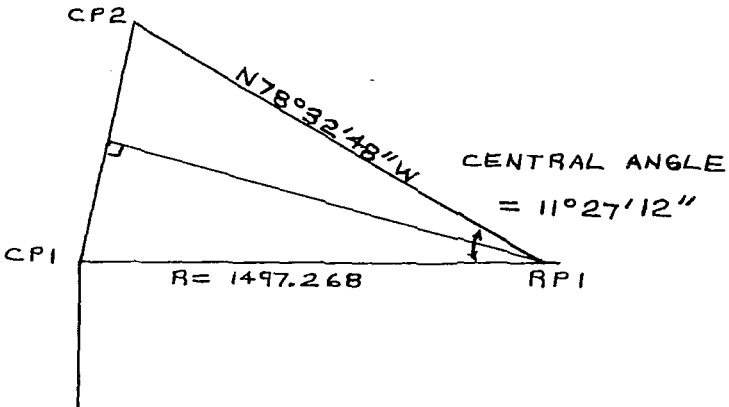
Step 6
Basically repeating steps 3 through 5.

Step 6A
Plot next control point. Determine bearing to last con-
trol point.

Step 6B
Strike a perpendicular bisector from this long chord to
intersection with the most previous radial line.

Step 6C
Determine these coordinates for next radius point.

Step 6D
Inverse between this radius point and the control point
of step 6A. This determines bearing and length of the
next radial line.

Step 6E
Determine central angle from difference between this and
most previous radius bearing. This next curve segment is
now defined.

Step 6F
Return to step 6A for each successive curve control point.

$$R_1 = 1497.268 \qquad R_2 = 421.686 \qquad R_3 = 1497.268$$
$$\Delta_1 = 11°27'12'' \qquad \Delta_2 = 67°05'36'' \qquad \Delta_3 = 11°27'12''$$

$$RP_1 = 50.000\,N \qquad RP_2 = 263.577\,N \qquad RP_3 = 790.576\,S$$
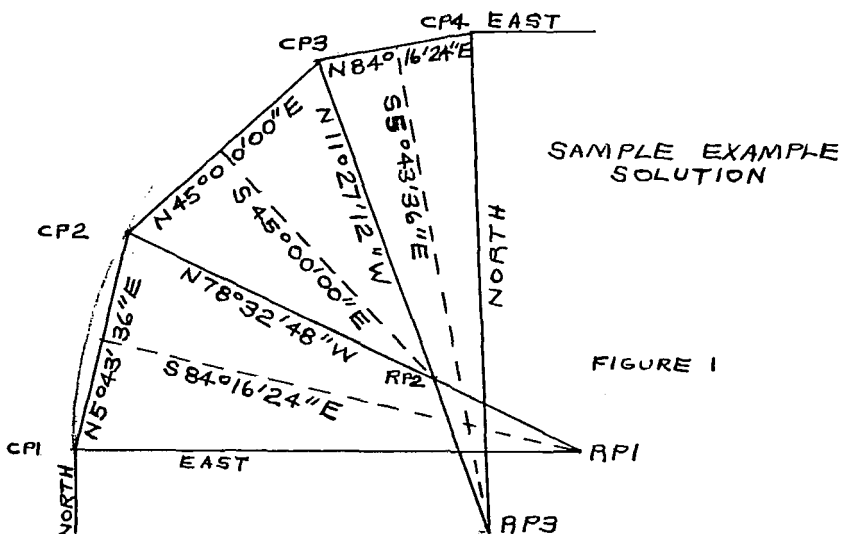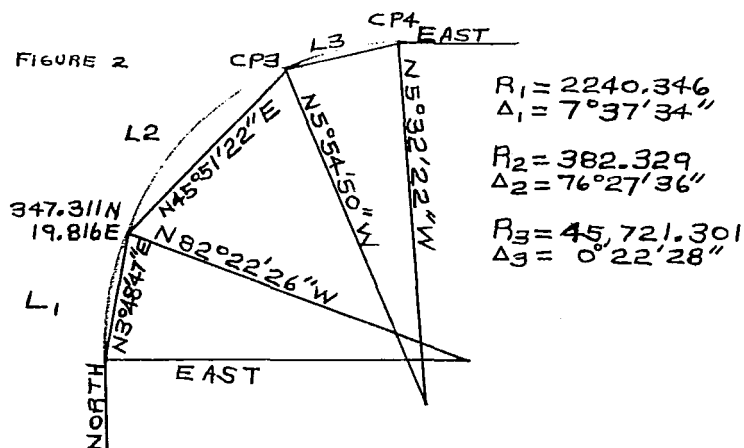$$1497.268\,E \qquad 443.105\,E \qquad 656.681\,E$$

SAMPLE EXAMPLE
SOLUTION

FIGURE 1

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The described method works well when all right-of-way
monuments are near record position. But what happens
when a monument is found well away from record?
Imagine a motor vehicle with no shock absorbers traveling
a dirt road. All is well until a sizable bump is in-
countered. The vehicle continues to bounce, and is
unable to track properly.

In much the same way, the flowcharted method of R-O-W
definition needs to be watched carefully after running
through an oddly positioned monument.

Following is the same example as before, but with the
effect of CP2 having been found 10 ft. west of its record
position in figure 1.

FIGURE 2

$R_1 = 2240.346$
$\Delta_1 = 7°37'34''$

$R_2 = 382.329$
$\Delta_2 = 76°27'36''$

$R_3 = 45,721.301$
$\Delta_3 = 0°22'28''$

Notice how after the "bump" the compound curve changes.
In a more extreme case, an undesired point of reverse
curvature could be created.

As the final leg of the example is a defined tangent,
and not a curve, our R-O-W definition gives a very poor
solution. Not only are the defined curves not very
representative of the intent of the original survey,
but the last radii is not perpendicular with the defined
exit tangent, a necessary condition for credibility.

A simple solution is to utilize the described method
from both the leading and exit tangents and approach the
"bump" from both directions. This will form a cusp at
the odd monument, but is certainly a more reasonable
solution.

Following is the "simple solution" for dealing with the
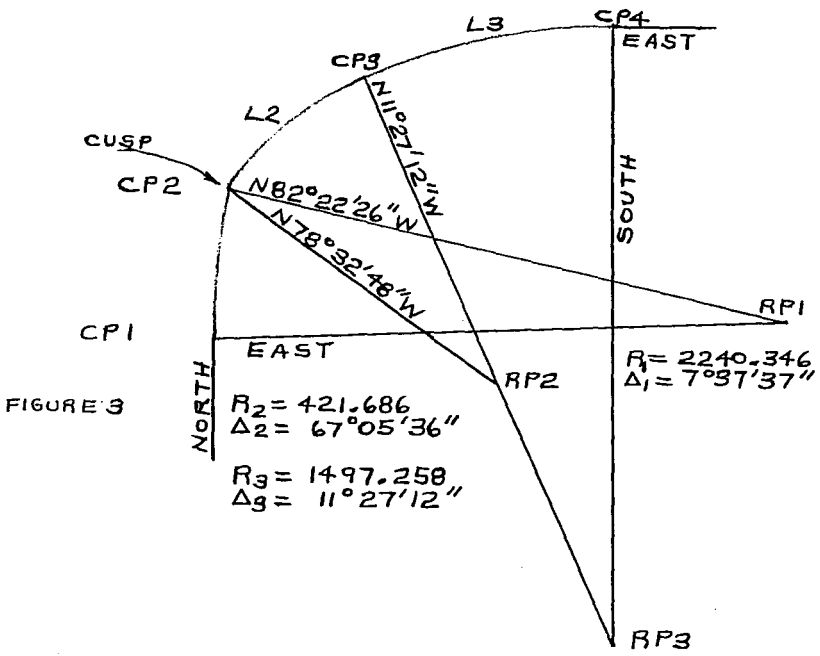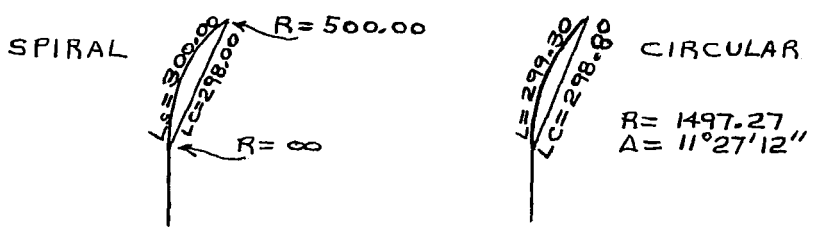above example's odd positioning of CP2.

FIGURE 3

Notice that the above solution is a combination of data
from figures 1 and 2.

How can the odd monument be determined? Most times this
will not be immediately apparent. Visual extrinsic
evidence such as a wild hook in a R-O-W fenceline may
give a clue. Comparison with the highway plat is also
a tool. When these methods fail, the resurveyor may
experiment with each R-O-W monument as his cusp point.
When the most reasonable solution is decided, the
right-of-way is defined.

AREA COMPARISONS

Sometimes the intent of the highway surveyor was to
create a spiral right-of-way. How does area adjacent
to a spiral curve compare with area adjacent to the
equivalent circular curve? Each case must be studied
independently. But, in example, comparing L1 from
figure 1 with the equivalent spiral curve

SPIRAL $R = 500.00$ $L = 300.00$ $LC = 298.00$ $R = \infty$

CIRCULAR $L = 299.30$ $LC = 298.80$ $R = 1497.27$ $\Delta = 11°27'12''$

thè spiral segment equals 2239 square feet while the
circular segment equals 1489 square feet. This differ-
ence is 50%, but only 0.02 acres.

If the entire example of figure 1, CP1 to CP4, was re-
placed by leading and exit spirals with dimensions of
that above, and a central circular curve R=500.00,
$\Delta=55^{\circ}33'28''$, the spiral system would have a segment area
of 154,506 ft.^{2} The area of the compound curve in
figure 1 is 157,100 ft.^{2} This difference is only 2%
or 0.06 acres.

CONCLUSION

Every solution method must be chosen by its application.
The described method for right-of-way definition by
circular curves is not different. In those situations
where land value is extremely high, boundaries unusually
critical, and plats call for spiral rights-of-way, use
of this method could be found inappropriate. For many
situations though, this method is adequate. Only the
land surveyor himself can make the decision.